arXiv:1209.3559v2 [hep-th] 22 Sep 2012

arXiv:1209.3559

Fermi-like Liquid From Einstein-DBI-Dilaton System

Shesansu Sekhar Pal

Centre of Excellence in Theoretical and Mathematical Sciences, SOA University, Bhubaneswar, 751030, India

shesansuATgmail.com

Abstract

We show the existence of a gravity dual of Fermi-like liquid in the classical limit. This we demonstrate by finding a Lifshitz type of black hole solution to the Einstein-DBI-Dilaton system in 3+1 dimensional spacetime. In particular, for $z = 2$, we show the linear and inverse quadratic temperature dependence of the specific heat and the longitudinal conductivity, respectively. However, we did not see the logarithmic structure of the entropy when the entangling region is of the strip type because the metric components does not obey the relation, $g^{d-2}_{xx}(r)g_{rr}(r) = 1/r^2$, where d is the dimension of the field theory. We also find a new class of dyonic black hole solution to the Einstein-DBI system in $3 + 1$ dimensional spacetime.

1 Introduction

There has been a lot of activity in trying to understand the scale invariant gravitational solution, which asymptotes to AdS at UV and at IR, it can behave either like AdS_2 or Lifshitz type. Recently, a non-scale invariant gravitational solution found in [1] and [2] has been interpreted to give the compressible state of the matter which exhibits the hidden Fermi surfaces [3], using holography [4]. In this context, it is suggested that the field theory directions and the invariant interval of the bulk scale in the following way

$$t \to \lambda^z t, \quad x_i \to \lambda^\delta x_i, \quad r \to \frac{r}{\lambda}, \quad ds \to \lambda^\gamma ds, \tag{1}$$

where z is the dynamical exponent, γ is the scaling violation exponent, which is related to the hyperscaling violation exponent as in [3], and more importantly, the spatial directions scale linearly, i.e., $\delta = 1$. See references [5]- [13] for further studies.

In this paper, we shall construct explicit solutions with $\gamma \neq 0$ along with $\gamma = 0$ and study its consequences. For $\gamma \neq 0$, we construct such a bulk solution with the help of gravity, U(1) gauge field and a scalar field for which the spatial directions do not scale, i.e., $\delta = 0$. In order to do so, let us consider a space filling brane, whose action is described by the Dirac-Born-Infeld (DBI) action. On considering the back reaction of the DBI action and that of the scalar field on to the geometry in 3+1 dimensional bulk spacetime, makes the metric looks as

$$ds^2 = r^2[-r^{2z}dt^2 + dx^2 + dy^2 + \frac{dr^2}{r^2}] \equiv r^2 ds_L^2. \tag{2}$$

It is easy to notice that the metric can be written as a spacetime which is conformal to the Lifshitz spacetime [14]. In which case, the Lifshitz spacetime scales as [15], [16],[17] and [18]

$$t \to \lambda^z t, \quad x_i \to \lambda^0 x_i, \quad r \to \frac{r}{\lambda}. \tag{3}$$

with $\gamma = -1$. See [1], [2] [19], [20] and [21] for an earlier study of the Einstein-Maxwell-Dilaton system.

It is argued in [3] that for a theory which exhibits the scaling violation exponent should see a reduced entropy. In fact for $d-1$ number of spatial directions with $\gamma = \theta/(d-1)$, the entropy should go as $S \sim T_H^{\frac{d-1-\theta}{z}}$, where T_H is the temperature. More importantly, in this case the spatial directions scale linearly, i.e., $x_i \to \lambda x_i$.

We show that for $\delta = 0$, there do not arise any changes in the entropy in contrast to the $\delta = 1$ case. In fact, the entropy behaves like that of the scale invariant solution. It is due to the fact that the Lifshitz solution can be written as an $AdS_2 \times R^2$ solution. Hence the entropy go as $S \sim T_H^{\frac{2}{z}}$. For this solution, see eq(28), the specific heat and the longitudinal conductivity goes as

$$c_v \sim T_H^{2/z}, \quad \sigma \sim T_H^{-2/z}. \tag{4}$$

We also show the existence of a Fermi-like liquid by doing an explicit computation of the transport and the thermodynamic quantity: the longitudinal conductivity as well as the specific heat. We found that the specific heat and the longitudinal conductivity has the linear and the inverse quadratic dependence on the temperature, respectively. This we demonstrate by finding an exact black hole solution to a $3+1$ dimensional Einstein-DBI-dilaton system. In which case the spacetime asymptotes to a Lifshitz spacetime with a non-trivial profile to the scalar field

$$ds^2 = -r^{2z}f(r)dt^2 + r^2(dx^2+dy^2) + \frac{dr^2}{r^2 f(r)}, \quad \phi(r) \sim log\ r, \quad f(r) = 1-(r_h/r)^{z+2} \tag{5}$$

and for some non-trivial form of the U(1) gauge field field strength such that it vanishes at IR in the zero temperature limit. In this case, the specific heat and the conductivity takes the following form

$$c_V \sim T^{2/z}, \quad \sigma \sim T^{-4/z}. \tag{6}$$

The Fermi liquid like behavior follows when the dynamical exponent takes a specific value, $z=2$. More importantly, the entropy in the zero temperature limit vanishes, suggesting the compressible nature of the configuration. Surprisingly, the computation of the entanglement entropy for a strip does not show up the necessary logarithmic term suggesting the absence of the Fermi surfaces. This finding in some sense supports the result of [22]. Further studies related to the Fermi liquid or the presence of Fermi surfaces are reported in [23]-[43].

It is interesting to note that for a specific value of the dynamically exponent, $z=2$, we see from eq(4) and eq(6) that only the conductivity go from the non-Fermi liquid (NFL) type to the Fermi liquid (FL) type. This happen for the choice of the parameters as defined in eq(12). This kind of "jump" from NFL phase to FL phase is noted previously in [55].

In the absence of the scalar field, we find an electrically charged black hole solution in arbitrary spacetime dimension at UV whose form precisely matches with that of the solution found for the Born-Infeld black holes in [46] but not the dyonic black hole solution in $3+1$ dimensional spacetime. For these type of black hole solutions there exits a non-zero entropy even at zero temperature. This particular property is similar in nature to that of the Reissner-Nordstrom (RN) black hole.

The findings of the paper for the Einstein-DBI-scalar field system in $d+1$ dimensional spacetime is summarized in Table (1).

The paper is organized as follows. In section 2, we write down the effective action and its equation of motion. In section 3, we shall present the solution at IR. In particular, the solution that shows the Fermi-like liquid behavior. In section 4, we find both the electrically charged black hole and a dyonic solution for trivial scalar field. Then show that the

Solutions at IR	Solutions at UV
For $g_{MN} \neq 0,\ F_{MN} \neq 0$ and $\phi = 0$, it generates $AdS_2 \times R^{d-1}$ Shows log structure in the entanglement entropy for $d = 2$	For $g_{MN} \neq 0,\ F_{MN} \neq 0$ and $\phi = 0$, (a) generates charged AdS black hole solution in any arbitrary spacetime dimensions; the entropy density in the vanishing temperature limit remains non-zero, $s = \frac{2\pi}{\kappa^2} T_b \frac{\rho}{\sqrt{4\Lambda^2 - T_b^2}}, \quad \text{as } T_H \to 0;$ The chemical potential is not a continuous function of the charge density. No Log structure in the entanglement entropy. (b) Dyonic AdS black hole solution in $3+1$ dimensional spacetime; the entropy density in the vanishing temperature limit remains non-zero, $s = \frac{2\pi}{\kappa^2} T_b \frac{\sqrt{\rho^2 + \lambda^2 B^2}}{\sqrt{4\Lambda^2 - T_b^2}}, \quad \text{as } T_H \to 0.$ No Log structure in the entanglement entropy.
For $g_{MN} \neq 0,\ F_{MN} \neq 0$ and $\phi \neq 0$, (a) Lifshitz solution with $\alpha \neq 0,\ \beta \neq 0$ in $3+1$ dimensional spacetime; the entropy density vanishes as temperature vanishes, $s \sim T_H^{2/z}$; The specific heat, $c_V \sim T_H^{2/z}$; The longitudinal conductivity, $\sigma \sim T_H^{-4/z}$; No Log structure in the entanglement entropy. (b) Hyper scaling violating solution but without decreasing the entropy with $\alpha \neq 0,\ \beta = 0$ in $3+1$ dimensional spacetime; the entropy density vanishes as temperature vanishes, $s \sim T_H^{2/z}$; The specific heat, $c_v \sim T_H^{2/z}$; The longitudinal conductivity, $\sigma \sim T_H^{-2/z}$; No Log structure in the entanglement entropy.	

Table 1: The summary of the solution of the Einstein-DBI-Dilaton system with two parameters α and β and the potential, $V(\phi)$, as defined in eq(12) and eq(13). The entanglement entropy is obtained for a strip.

electrically charged black hole solution found for the DBI actions are same as that found for Born-Infeld actions. In section 5 and 6, we compute the conductivity as well as the entanglement entropy, respectively. And finally we conclude in section 7.

2 The action

The action that we consider contains metric, the abelian gauge field and the scalar field as the degrees of freedom. In particular, the action involving the gauge field is the non-linear generalization of the Maxwell action, namely the Dirac-Born-Infeld action. The exact form of the action is[1]

$$S=\frac{1}{2\kappa^2}\int d^{d+1}x\left[\sqrt{-g}\left(R-2\Lambda-\frac{1}{2}\partial_M\phi\partial^M\phi-V(\phi)\right)-T_bZ_1(\phi)\sqrt{-det\Big([g]Z_2(\phi)+\lambda F\Big)_{ab}}\right], \tag{7}$$

where $[g]_{ab}=\partial_aX^M\partial_bX^Ng_{MN}$ is the induced metric on to the world volume of the brane. T_b and Λ are the tension of the brane, and cosmological constant, respectively. $F=dA$ is the two-form field strength. Since, we are considering the brane to fill the entire space, means $[g]_{ab}=g_{ab}$.

The equation of motion of the metric component that follows from it takes the following form

$$\begin{aligned}&R_{MN}-\frac{2\Lambda}{(d-1)}g_{MN}-\frac{g_{MN}}{(d-1)}V(\phi)-\frac{1}{2}\partial_M\phi\partial_N\phi-\\&\frac{T_b\ Z_1(\phi)Z_2(\phi)}{4(d-1)}\frac{\sqrt{-det\Big(g\ Z_2(\phi)+\lambda F\Big)_{ab}}}{\sqrt{-g}}\left[\Big(g\ Z_2(\phi)+\lambda F\Big)^{-1}+\Big(g\ Z_2(\phi)-\lambda F\Big)^{-1}\right]^{KL}\\&\Big[g_{MN}g_{KL}-(d-1)g_{MK}g_{NL}\Big]=0.\end{aligned} \tag{8}$$

The gauge field equation of motion is

$$\partial_M\left[Z_1(\phi)\sqrt{-det\Big(g\ Z_2(\phi)+\lambda F\Big)_{ab}}\left(\Big(g\ Z_2(\phi)+\lambda F\Big)^{-1}-\Big(g\ Z_2(\phi)-\lambda F\Big)^{-1}\right)^{MN}\right]=0 \tag{9}$$

It follows trivially that the gauge field can be fully determined in terms of the metric components and the dilaton. Finally, the equation of motion of the scalar field

$$\begin{aligned}&\partial_M\Big(\sqrt{-g}\partial^M\phi\Big)-\sqrt{-g}\frac{dV(\phi)}{d\phi}-T_b\frac{dZ_1(\phi)}{d\phi}\sqrt{-det\Big(g\ Z_2(\phi)+\lambda F\Big)_{ab}}-\\&\frac{T_b}{2}Z_1(\phi)\sqrt{-det\Big(g\ Z_2(\phi)+\lambda F\Big)_{ab}}\frac{dZ_2(\phi)}{d\phi}\Big(g\ Z_2(\phi)+\lambda F\Big)^{-1MN}g_{MN}=0.\end{aligned} \tag{10}$$

[1]See [64], where the authors used a related action to study the holographic QCD in the Veneziano limit.

Let us consider an ansatz where the metric, the abelian field strength and the dilaton to be of the following form

$$ds^2_{d+1} = -g_{tt}(r)dt^2 + g_{rr}(r)dr^2 + g_{xx}(r)dx_i^2, \quad A = A_t(r)dt, \quad F = A_t' dr \wedge dt, \quad \phi = \phi(r) \tag{11}$$

with the following form of the functions

$$Z_1(\phi) = exp(-\alpha\phi), \quad Z_2(\phi) = exp(\beta\phi), \tag{12}$$

and we choose the potential as

$$V(\phi) = m_1 \; exp(m_2 \; \phi), \tag{13}$$

where m_1 and m_2 are constants. With this structure of the ansatz, there exists several exact solutions.

Given such a choice of the metric as written in eq(11), the various non-vanishing components of the Ricci tensor are

$$\begin{aligned}
R_{tt} &= \frac{g_{tt}''}{2g_{rr}} + (d-1)\frac{g_{tt}'g_{xx}'}{4g_{rr}g_{xx}} - \frac{g_{tt}'^2}{4g_{rr}g_{tt}} - \frac{g_{tt}'g_{rr}'}{4g_{rr}^2}, \\
R_{ij} &= \delta_{ij}\Big[-\frac{g_{xx}''}{2g_{rr}} - (d-3)\frac{g_{xx}'^2}{4g_{rr}g_{xx}} + \frac{g_{xx}'g_{rr}'}{4g_{rr}^2} - \frac{g_{tt}'g_{xx}'}{4g_{rr}g_{tt}}\Big], \\
R_{rr} &= -(d-1)\frac{g_{xx}''}{2g_{xx}} - \frac{g_{tt}''}{2g_{tt}} + (d-1)\frac{g_{xx}'^2}{4g_{xx}^2} + (d-1)\frac{g_{rr}'g_{xx}'}{4g_{rr}g_{xx}} + \frac{g_{tt}'^2}{4g_{tt}^2} + \frac{g_{tt}'g_{rr}'}{4g_{rr}g_{tt}}
\end{aligned} \tag{14}$$

Solving the equation of motion of the gauge field gives

$$\lambda A_t' = \frac{\rho Z_2\sqrt{g_{tt}g_{rr}}}{\sqrt{\rho^2 + Z_1^2 Z_2^{d-1} g_{xx}^{d-1}}}, \quad \Longrightarrow g_{tt}g_{rr}Z_2^2 - \lambda^2 A_t'^2 = \frac{g_{tt}g_{rr}g_{xx}^{(d-1)}Z_1^2 Z_2^{d+1}}{\rho^2 + Z_1^2 Z_2^{d-1} g_{xx}^{d-1}} \tag{15}$$

where ρ is the constant of integration and interpreted as the charge density. The equation of motion of the scalar field can be simplified as

$$\begin{aligned}
&\partial_r\Big(\sqrt{\frac{g_{tt}}{g_{rr}}}g_{xx}^{(d-1)/2}\phi'\Big) - \frac{dV}{d\phi}\sqrt{g_{tt}g_{rr}}\; g_{xx}^{(d-1)/2} - \Big[\frac{dZ_1}{d\phi} + \frac{Z_1}{Z_2}\Big(\frac{d-1}{2}\Big)\frac{dZ_2}{d\phi}\Big] \times \\
&T_b Z_2^{(d-1)/2} g_{xx}^{(d-1)/2}\sqrt{g_{tt}g_{rr}Z_2^2 - \lambda^2 A_t'^2} - T_b Z_1 \frac{Z_2^{(d+1)/2} g_{tt}g_{rr}g_{xx}^{(d-1)/2}}{\sqrt{g_{tt}g_{rr}Z_2^2 - \lambda^2 A_t'^2}}\frac{dZ_2}{d\phi} = 0.
\end{aligned} \tag{16}$$

Now using the solution of the gauge field, the equation of motion of the scalar field becomes

$$\begin{aligned}
&\partial_r\Big(\sqrt{\frac{g_{tt}}{g_{rr}}}g_{xx}^{(d-1)/2}\phi'\Big) - \frac{dV}{d\phi}\sqrt{g_{tt}g_{rr}}\; g_{xx}^{(d-1)/2} - T_b\frac{dZ_2}{d\phi}\sqrt{g_{tt}g_{rr}}\sqrt{\rho^2 + Z_1^2 Z_2^{d-1} g_{xx}^{d-1}} \\
&-\Big[\frac{dZ_1}{d\phi} + \frac{Z_1}{Z_2}\Big(\frac{d-1}{2}\Big)\frac{dZ_2}{d\phi}\Big]\Bigg(\frac{T_b\sqrt{g_{tt}g_{rr}}g_{xx}^{d-1}Z_1 Z_2^d}{\sqrt{\rho^2 + Z_1^2 Z_2^{d-1} g_{xx}^{d-1}}}\Bigg) = 0.
\end{aligned} \tag{17}$$

Finally, the equation of motion of the metric component can be expressed, explicitly, as follows

$$R_{tt}+\frac{V+2\Lambda}{d-1}g_{tt}-T_b\frac{(d-3)}{2(d-1)}\frac{Z_2 g_{tt}}{g_{xx}^{(d-1)/2}}\sqrt{\rho^2+Z_1^2Z_2^{d-1}g_{xx}^{d-1}}+\frac{T_b}{2}\frac{Z_1^2Z_2^d g_{tt}g_{xx}^{(d-1)/2}}{\sqrt{\rho^2+Z_1^2Z_2^{d-1}g_{xx}^{d-1}}}=0, \tag{18}$$

$$R_{ij}-\frac{V+2\Lambda}{d-1}g_{xx}\delta_{ij}-T_b\frac{\delta_{ij}}{d-1}\frac{Z_2\sqrt{\rho^2+Z_1^2Z_2^{d-1}g_{xx}^{d-1}}}{g_{xx}^{(d-3)/2}}=0, \tag{19}$$

$$R_{rr}-\frac{V+2\Lambda}{d-1}g_{rr}-\frac{1}{2}\phi'^2+T_b\frac{(d-3)g_{rr}}{2(d-1)}\frac{Z_2\sqrt{\rho^2+Z_1^2Z_2^{d-1}g_{xx}^{d-1}}}{g_{xx}^{(d-1)/2}}-\frac{T_b}{2}Z_1^2Z_2^d\frac{g_{rr}g_{xx}^{(d-1)/2}}{\sqrt{\rho^2+Z_1^2Z_2^{d-1}g_{xx}^{d-1}}}=0 \tag{20}$$

3 Exact solution at IR: $AdS_2\times R^{d-1}$

In a special case for which the potential energy is trivial, $V(\phi)=0$, along with constant Z_1 and Z_2, i.e., $Z_1=1=Z_2$, the dilaton can be taken as trivial. In which case, it is expected that the solution near the IR end should take the following form $AdS_2\times R^{d-1}$ and the explicit form of it looks as

$$ds^2_{d+1}=-\frac{r^2}{R_2^2}dt^2+\frac{R_2^2}{r^2}dr^2+c_0^2\delta_{ij}dx^idx^j,\quad A=\frac{e_d}{R_2^2}r\ dt, \tag{21}$$

where R_2 is the size of the AdS_2 spacetime and we have set $\lambda=1$ for convenience. The tension of the brane and the cosmological constant is determined as

$$T_b=\frac{2}{e_d^2}\sqrt{R_2^4-e_d^2},\quad \Lambda=-\frac{T_bR_2^2}{2\sqrt{R_2^4-e_d^2}}=-\frac{R_2^2}{e_d^2}. \tag{22}$$

It is easy to notice that for real valued tension, T_b, the brane requires the constraint $R_2^4\geq e_d^2$ and such a condition is easily met by looking at the equation of motion of the gauge field. The constant e_d is determined in terms of the charge density, ρ, as

$$e_d=\frac{\rho\ R_2^2}{\sqrt{\rho^2+c_0^{2(d-1)}}}. \tag{23}$$

The finite temperature solution at IR with only non-zero electric field in any arbitrary $d+1$ spacetime dimensions

$$ds^2_{d+1} = -\frac{r^2}{R_2^2}\left(1-\frac{r_h}{r}\right)dt^2 + \frac{R_2^2}{r^2\left(1-\frac{r_h}{r}\right)}dr^2 + c_0^2\delta_{ij}dx^i dx^j, \quad A = \frac{\rho\ (r-r_h)}{\sqrt{c_0^{2(d-1)}+\rho^2}}\ dt, \tag{24}$$

with the tension of the brane and the cosmological constant as written in eq(22).

If we want to turn on a constant magnetic field along with an electric field for which the 1-form gauge potential takes the following form $A = \frac{e_d}{R_2^2}rdt + \frac{B}{2}(x_1dx_2 - x_2dx_1)$, then the finite temperature solution at IR, let us say in $3+1$ spacetime dimensions, takes the following form

$$ds^2 = -\frac{r^2}{R_2^2}\left(1-\frac{r_h}{r}\right)dt^2 + \frac{R_2^2}{r^2\left(1-\frac{r_h}{r}\right)}dr^2 + c_0^2(dx^2+dy^2), \quad A = \frac{\rho\ (r-r_h)}{\sqrt{c_0^4+B^2+\rho^2}}\ dt. \tag{25}$$

The tension of the brane and the cosmological constant takes the following form

$$T_b = \frac{2c_0^2\sqrt{c_0^4+B^2+\rho^2}}{R_2^2(B^2+\rho^2)}, \quad \Lambda = -\frac{c_0^4+B^2+\rho^2}{R_2^2(B^2+\rho^2)}. \tag{26}$$

3.1 A black hole solution for $\beta = 0$

In $3+1$ dimensional bulk spacetime dimension, there exists a black hole solution at IR, which is conformal to the Lifshitz spacetime (or to $AdS_2 \times R^2$). In order to construct such a black hole solution, we choose the potential as

$$V(\phi) = m_1\ exp(m_2\ \phi), \tag{27}$$

where m_1 and m_2 are constants. The solution reads as

$$\begin{aligned}
ds^2 &= r^2[-r^{2z}f(r)dt^2 + dx^2 + dy^2 + \frac{dr^2}{r^2 f(r)}], \quad F = \frac{\rho\ r^{z+1}}{\sqrt{\rho^2\lambda^2 + exp(-\frac{2\phi_0}{\sqrt{z+1}})}}dr\wedge dt \\
\phi &= \phi_0 + 2\sqrt{z+1}Log\ r, \quad m_1 = -\frac{2(2+z)exp(-\frac{\phi_0}{\sqrt{z+1}})}{\rho^2\lambda^2}[z + (z+1)\rho^2\lambda^2 exp(2\frac{\phi_0}{\sqrt{z+1}})], \\
T_b &= 2\frac{e^{-\frac{\phi_0}{\sqrt{z+1}}}(z^2+2z)}{\rho^2\lambda^2}\sqrt{1+\rho^2\lambda^2 exp(\frac{2\phi_0}{\sqrt{z+1}})}, \quad \alpha = \frac{1}{\sqrt{z+1}}, \quad \beta = 0, \quad \Lambda = 0 \\
m_2 &= 2\beta - \alpha = -\frac{1}{\sqrt{z+1}}, \quad f(r) = 1 - \left(\frac{r_h}{r}\right)^{2+z}, \quad Z_1 = \frac{e^{-\alpha\phi_0}}{r^2}, \quad Z_2 = 1,
\end{aligned} \tag{28}$$

where ϕ_0 is a constant and z is the dynamical exponent. Even though we have set the quantity Λ to zero it does not mean the cosmological constant vanishes. In fact upon expanding the potential, $V(\phi)$, one sees the presence of a negative cosmological constant, which is determined by m_1. The constant ρ is the charge density.

Let us calculate the Hawking temperature associated to the black hole solution as written in eq(28). It is calculated from the following formula

$$\kappa^2 = -\frac{1}{2}\nabla^a\varepsilon^b\nabla_a\varepsilon_b, \quad T_H = \frac{\kappa}{2\pi}, \tag{29}$$

where the null vector ε^a defines the horizon, $(\varepsilon^a\varepsilon_a)_{r_h} = 0$ and the temperature, T_H, is evaluated on the horizon. For a spacetime of the form: $ds^2_{d+1} = -g_{tt}(r)dt^2 + g_{rr}(r)dr^2 + g_{xx}(r)dx^i dx_i$, the Hawking temperature of the system essentially becomes

$$T_H = \frac{1}{4\pi}\left(\frac{g'_{tt}}{\sqrt{g_{tt}g_{rr}}}\right)_{r_h}, \tag{30}$$

where prime denotes derivative with respect to the radial coordinate, r. Doing the calculation for the solution eq(28), we find the temperature as

$$T_H = \frac{2+z}{4\pi} r_h^z. \tag{31}$$

The entropy density which is the area of the horizon divided by $4G = \frac{\kappa^2}{2\pi}$ gives

$$s = \frac{1}{4G}\left(\frac{4\pi}{2+z}\right)^{\frac{2}{z}} T_H^{\frac{2}{z}}. \tag{32}$$

It is interesting to note that the entropy density vanishes as the temperature vanishes, for positive dynamical exponent. There follows, the specific heat, $c_v = T_H\left(\frac{\partial s}{\partial T_H}\right)_\rho \sim T_H^{2/z}$.

3.1.1 Null Energy Condition

Given the choice of our action in eq(7), the energy-momentum tensor takes the following form

$$\begin{aligned} T_{MN} &= \partial_M\phi\partial_N\phi - g_{MN}\left[V + 2\Lambda + \frac{1}{2}(\partial\phi)^2\right] - \\ &\quad \frac{T_b}{2\sqrt{-g}} Z_1 Z_2 \sqrt{-det(Z_2 g + \lambda F)}\Big[(Z_2 g + \lambda F) + (Z_2 g - \lambda F)\Big]^{KL} g_{KM}g_{NL} \end{aligned} \tag{33}$$

Demanding that the system that we are dealing with should satisfy the null energy condition, $T_{MN}u^M u^N \geq 0$ for some null vectors u^M gives us the restriction on the Ricci

tensor as $R_{MN}u^M u^N \geq 0$. By considering the two possible choices for the null vectors as $u^t = 1/\sqrt{g_{tt}},\ u^r = 1/\sqrt{g_{rr}}, u^i = 0$ and $u^t = 1/\sqrt{g_{tt}},\ u^{x_1x_1} = 1/\sqrt{g_{x_1x_1}},\ u^r = 0$ and setting the rest of the vectors to zero, gives the following conditions for the metric of the type

$$ds^2_{d+1} = -r^{2(z+1)}dt^2 + r^2 dx^i dx_i + dr^2, \quad \Rightarrow (z+1)(d-1) \geq 0, \quad z(d+z-1) \geq 0. \tag{34}$$

We obtain such a form of the metric in the zero temperature limit of eq(28). Upon solving the inequality, we find the most interesting restriction that is

$$d \geq 2 \quad z \geq 0, \tag{35}$$

whereas the other possibilities are not that interesting because either the dimensionality of the spacetime or the dynamical exponent could become negative.

3.2 Lifshitz solution: $(\alpha \neq 0, \beta \neq 0)$

In order to generate a Lifshitz solution, we shall consider the case where all the degrees of freedom are non-trivial i.e., they do not vanish, as well as the functions Z_1 and Z_2 are not set to unity. But we shall take a trivial potential energy, $V = 0$ with non-zero cosmological constant, $\Lambda \neq 0$. For simplicity, we shall be solving the equations of motion in $3+1$ dimensional bulk spacetime dimensions. In this case the solution reads as

$$\begin{aligned} ds^2_{3+1} &= -r^{2z}f(r)dt^2 + r^2(dx_1^2 + dx_2^2) + \frac{dr^2}{r^2 f(r)}, \quad f(r) = 1 - \frac{r_h^{z+2}}{r^{z+2}}, \\ \phi(r) &= 2\sqrt{z-1}\ Log\ r, \quad Z_1 = \frac{1}{r^4}, \quad Z_2 = r^2, \quad F = \frac{\rho r^{z+1}}{\sqrt{1+\lambda^2\rho^2}} dr \wedge dt \\ T_b &= 2\frac{\sqrt{1+\lambda^2\rho^2}}{\lambda^2\rho^2}(z^2+z-2), \quad \Lambda = -\frac{z^2(1+\lambda^2\rho^2) + z(1+2\lambda^2\rho^2) - 2}{\lambda^2\rho^2}, \end{aligned} \tag{36}$$

where the dynamical exponent z should always be bigger than unity, $z > 1$. The Hawking temperature, in this case, turns out to be

$$T_H = \frac{(z+2)}{4\pi} r_h^z. \tag{37}$$

With the Bekenstein-Hawking entropy density given as $s = \kappa^2/(2\pi)\ (\frac{4\pi}{(z+2)})^{2/z} T_H^{2/z}$. From this expression of the entropy density, it follows trivially that for positive dynamical exponent the entropy vanishes as temperature vanishes. The specific heat, $c_v = T_H \left(\frac{\partial s}{\partial T_H}\right)_\rho \sim T_H^{2/z}$. It is interesting to note that for a specific choice of the dynamical exponent, $z = 2$, the specific heat has a linear temperature dependence.

4 Black hole solution at UV

Let us construct a black hole solution for a specific choice of the functions that appear in the action eq(7), namely, we set $Z_1 = 1 = Z_2$ and the potential energy as $V = 0$. For this choice of the functions, it follows that the solution to the scalar field can be taken as trivial i.e., $\phi = 0$, in which case the gauge field takes the following form

$$\lambda A_t' = \frac{\rho\sqrt{g_{tt}g_{rr}}}{\sqrt{\rho^2 + g_{xx}^{d-1}}}. \tag{38}$$

The equation of motion of the metric component reduces to

$$R_{MN} - \frac{2\Lambda}{(d-1)}g_{MN} - \frac{T_b}{4(d-1)}\frac{\sqrt{-det\Big(g+\lambda F\Big)_{ab}}}{\sqrt{-g}}\Big[\Big(g+\lambda F\Big)^{-1} + \Big(g-\lambda F\Big)^{-1}\Big]^{KL}$$
$$\Big[g_{MN}g_{KL} - (d-1)g_{MK}g_{NL}\Big] = 0. \tag{39}$$

Let us assume that the geometry, asymptotically, approach the AdS spacetime. We consider the following form of the spacetime for explicit calculations

$$ds_{d+1}^2 = \frac{r^2}{R^2}[-f(r)dt^2 + dx_i^2] + \frac{R^2dr^2}{r^2f(r)}, \tag{40}$$

where R is the size of the AdS spacetime. Let us substitute this ansatz into the equations of motion of the metric eq(39), then there arises two second order differential equations. One from the g_{tt} and the other from the g_{rr} component. In fact these two differential equations are not independent, the precise relation is $\frac{2R^4}{r^2f}\times$ eq(18) $= -2r^2\times$ eq(20). So we left with only one second order differential equation, which reads as

$$r^2f'' + (d+3)rf' + 2df + \frac{4\Lambda R^2}{d-1} + \frac{T_bR^2(r/R)^{d-1}}{\sqrt{\rho^2 + \frac{r^{2(d-1)}}{R^{2(d-1)}}}} - \frac{d-3}{d-1}T_bR^2\frac{r^{d-1}}{R^{d-1}}\sqrt{\rho^2 + \frac{r^{2(d-1)}}{R^{2(d-1)}}} = 0. \tag{41}$$

Now, this equation can be reduced to a first order differential equation, which essentially follows from eq(19) and the precise relation is $-\frac{R^4}{r}\partial_r(eq(19)) =$ eq(41). Finally, the equation of motion that follows from eq(19)

$$rf'(r) + df(r) + \frac{2\Lambda R^2}{d-1} + \frac{T_bR^2}{d-1}\frac{r^{1-d}}{R^{1-d}}\sqrt{\rho^2 + \frac{r^{2(d-1)}}{R^{2(d-1)}}} = 0. \tag{42}$$

On solving this differential equation for the generic choice of the dimension gives

$$f(r) = \frac{c_1}{r^d} - \frac{2\Lambda R^2}{d(d-1)} - \frac{T_bR^2\rho}{(d-1)}\frac{r^{1-d}}{R^{1-d}}\ {}_2F_1\Big[-\frac{1}{2}, \frac{1}{2(d-1)}, \frac{2d-1}{2(d-1)}, -\frac{r^{2(d-1)}}{R^{2(d-1)}\rho^2}\Big]. \tag{43}$$

To get a feel of the solution, in what follows, we shall try to solve it for few specific choices of the spacetime dimension. Let us assume that the geometry asymptotes to AdS_3, i.e., we set $d=2$, in which case the solution is

$$
\begin{aligned}
ds_3^2 &= \frac{r^2}{R^2}[-f(r)dt^2+dx^2]+\frac{R^2dr^2}{r^2f(r)}, \quad \text{with} \\
f(r) &= \frac{c_1}{r^2}-\Lambda R^2-T_bR^2\frac{\sqrt{r^2+R^2\rho^2}}{2r}-\frac{R^4T_b\rho^2}{2r^2}Log\Big(r+\sqrt{r^2+R^2\rho^2}\Big),
\end{aligned} \tag{44}
$$

where c_1 is a constant. The horizon, r_h, is determined as the location for which $f(r_h)=0$. For AdS_4, the solution looks as

$$
\begin{aligned}
ds_4^2 &= \frac{r^2}{R^2}[-f(r)dt^2+dx_1^2+dx_2^2]+\frac{R^2dr^2}{r^2f(r)}, \\
f(r) &= \frac{c_1}{r^3}-\frac{1}{3}R^2\Lambda-\frac{1}{6r^2}T_bR^2\sqrt{r^4+R^4\rho^2}-\frac{1}{3r^2}T_b\rho R^4\ {}_2F_1[\frac{1}{4},\frac{1}{2},\frac{5}{4},-\frac{r^4}{R^4\rho^2}].
\end{aligned} \tag{45}
$$

Similarly, solving for an AdS_5 spacetime, we find

$$
\begin{aligned}
ds_5^2 &= \frac{r^2}{R^2}[-f(r)dt^2+dx_1^2+dx_2^2+dx_3^2]+\frac{R^2dr^2}{r^2f(r)}, \\
f(r) &= \frac{c_1}{r^4}-\frac{1}{6}R^2\Lambda-\frac{1}{12r^3}T_bR^2\sqrt{r^6+R^6\rho^2}-\frac{1}{4r^3}T_b\rho R^5\ {}_2F_1[\frac{1}{6},\frac{1}{2},\frac{7}{6},-\frac{r^6}{R^6\rho^2}],
\end{aligned} \tag{46}
$$

where ${}_2F_1[a,b,c,x]$ is the hypergeometric function. In order to fix the constant, c_1, we need to do an expansion in the small charge density, ρ, limit and compare it with the RN black hole solution. From which we can identify the constant c_1 as the mass of the black hole, $c_1\propto -M$. The explicit calculation is presented towards the end of this section.

The gauge field that supports the AdS spacetime, from eq(38), follows as

$$
\lambda A_t(r)=r\ R^{d-1}\ {}_2F_1\left[\frac{1}{2},\frac{1}{2(d-1)},\frac{2d-1}{2(d-1)},-\frac{(r/R)^{2(d-1)}}{\rho^2}\right]+\Phi. \tag{47}
$$

The constant quantity, Φ, is determined by requiring that the gauge field should vanish at the horizon, $A_t(r_h)=0$, in order to keep the norm of the gauge potential finite at the horizon. The chemical potential is determined by

$$
\begin{aligned}
\mu &= \int_{r_h}^{\infty} A_t' = \frac{1}{\lambda}\int_{r_h}^{\infty}\frac{\rho R^{d-1}}{\sqrt{r^{2(d-1)}+\rho^2R^{2(d-1)}}} \\
&= \frac{1}{\lambda}\left(\frac{\rho_t^{\frac{1}{d-1}}}{(d-2)\sqrt{\pi}}\Gamma\Big(\frac{4-3d}{2-2d}\Big)\Gamma\Big(\frac{1}{2d-2}\Big)-r_h\ {}_2F_1\left[\frac{1}{2},\frac{1}{2(d-1)},\frac{1-2d}{2-2d},-\frac{r_h^{2(d-1)}}{\rho_t^2}\right]\right),
\end{aligned} \tag{48}
$$

where $\rho_t \equiv \rho R^{d-1}$. This particular form of the gauge potential, hence the chemical potential, matches precisely with the one computed in the probe approximation in [44] i.e., without taking the back reaction of the gauge field onto the geometry. We can determine whether this particular state corresponds to a compressible phase or not by simple looking at the continuity of the chemical potential with respect to the charge density.

$$\begin{aligned}\frac{d\mu}{d\rho} &= \frac{R}{\lambda(d-1)^2}\Bigg(\frac{\rho^{\frac{2-d}{d-1}}}{\sqrt{\pi}}\Gamma\Big(\frac{4-3d}{2-2d}\Big)\Gamma\Big(\frac{1}{2d-2}\Big)+\frac{r_h\rho(d-1)}{\sqrt{\rho^2-R^{2-2d}r_h^{2d-2}}}\\ &\quad -r_h(d-1)\ {}_2F_1\left[\frac{1}{2},\frac{1}{2(d-1)},\frac{1-2d}{2-2d},\frac{r_h^{2(d-1)}R^{2(1-d)}}{\rho^2}\right]\Bigg). \qquad (49)\end{aligned}$$

At a very specific value of the charge density, namely, $\rho = (r_h/R)^{d-1}$, the above derivative has a singularity. So, we conclude that the dual field theory of the Einstein-DBI system does not show up the necessary feature to be part of the compressible phase of matter.

The temperature of the $d+1$ dimensional black hole can be computed from the formula as written in eq(30) as

$$T_H = -\frac{r_h}{(d-1)4\pi R^2}\left[2\Lambda + T_b\ r_h^{1-d}\sqrt{\rho^2 R^{2(d-1)} + r_h^{2(d-1)}}\right], \qquad (50)$$

where we have used eq(42) to find the derivative of the function $f(r)$. The Bekenstein-Hawking entropy density becomes

$$s = \frac{2\pi}{\kappa^2}\left(\frac{r_h}{R}\right)^{d-1}. \qquad (51)$$

Let us find the entropy in a limit for which the temperature of the black hole vanishes, $T_H = 0$, for a non-zero size of the horizon, $r_h \neq 0$. This happen when the size of the horizon takes the following form

$$r_h^{d-1} = \frac{T_b \rho R^{d-1}}{\sqrt{4\Lambda^2 - T_b^2}}. \qquad (52)$$

In this case the entropy density becomes

$$s_{ext} = \frac{2\pi}{\kappa^2}\frac{T_b\rho}{\sqrt{4\Lambda^2 - T_b^2}} \quad \neq 0. \qquad (53)$$

It means even for the non-linearly generalized Einstein-Maxwell action that is the Einstein-DBI action has a non-zero entropy at zero temperature. Moreover, the existence of non-zero entropy or the non-zero horizon size at zero temperature suggests an upper bound on the tension of the brane $T_b^2 < 4\Lambda^2$ and is consistent with the solution found in eq(22) for AdS_2 .

The specific heat $C_V=T_H\left(\frac{\partial s}{\partial T_H}\right)_\rho$ that follows

$$C_V=\frac{2(d-1)\pi R^{3-d}r_h^d\sqrt{\rho^2R^{2(d-1)}+r_h^{2(d-1)}}[2\Lambda r_h^d+T_br_h\sqrt{\rho^2R^{2(d-1)}+r_h^{2(d-1)}}]}{\kappa^2[R^2r_h^d(T_br_h^d+2\Lambda r_h\sqrt{\rho^2R^{2(d-1)}+r_h^{2(d-1)}})-(d-2)T_b\rho^2R^{2d}r_h^2]} \tag{54}$$

In our notation T_b is positive and we are dealing with spacetimes of negative cosmological constant, which means there exists a range of values of the charge density for which the solution has got positive specific heat. In this range of charge densities, the system is thermodynamically stable. In fact, for a choice like, $d=3,\ R=1=r_h$, the specific heat, $C_V=\frac{4\pi\sqrt{1+\rho^2}[2\Lambda+T_b\sqrt{1+\rho^2}]}{T_b(1-\rho^2)+2\Lambda\sqrt{1+\rho^2}}$. For small charge density, it becomes $C_V=4\pi+\frac{8\pi T_b}{T_b+2\Lambda}\rho^2+\mathcal{O}(\rho)^4$.

Let us fix the precise relation between the constant c_1 and the mass M, in order to do so, let us use the thermodynamic relation $dM=T_HdS+\Phi d\rho$. Since the charge density is constant means the mass of the black hole can be found from $M=\int dr_hT_H\left(\frac{\partial S}{\partial r_h}\right)$. Doing the above integral along with the use of the following relations for Hypergeometric functions

$$(a-b)\ {}_2F_1[a,b,c,x]=a\ {}_2F_1[1+a,b,c,x]-b\ {}_2F_1[a,1+b,c,x],\quad {}_2F_1[a,b,b,x]=(1-x)^{-a}, \tag{55}$$

gives the mass as

$$-2\kappa^2R^{d+1}M=\frac{2\Lambda}{d}r_h^d+T_b\rho R^{d-1}r_h\ {}_2F_1\left[-\frac{1}{2},\frac{1}{2(d-1)},\frac{1-2d}{2-2d},-\frac{r_h^{2(d-1)}}{\rho^2R^{2(d-1)}}\right]. \tag{56}$$

Recall that the constant c_1 is determined from the condition, $f(r_h)=0$, which means

$$c_1=\frac{2\Lambda R^2r_h^d}{d(d-1)}+\frac{T_b\rho R^{d+1}}{(d-1)}r_h\ {}_2F_1\left[-\frac{1}{2},\frac{1}{2(d-1)},\frac{1-2d}{2-2d},-\frac{r_h^{2(d-1)}}{\rho^2R^{2(d-1)}}\right]. \tag{57}$$

On comparing these two expressions, we find $c_1=-\frac{2\kappa^2}{(d-1)}R^{d+3}M$.

4.1 Dyonic solution

In this case we turn on both the electric field along with a magnetic field in $3+1$ spacetime dimensions. The magnetic field is considered to be a constant, for simplicity. Hence the explicit structure of the field strength and the metric is

$$F=A_t'(r)dr\wedge dt+Bdx\wedge dy,\quad ds_{3+1}^2=-g_{tt}(r)dt^2+g_{xx}(r)(dx^2+dy^2)+g_{rr}(r)dr^2. \tag{58}$$

Let us solve the equation of motion associated to the gauge field and is given as

$$\lambda A_t'=\frac{\rho Z_2\sqrt{g_{tt}g_{rr}}}{\sqrt{\rho^2+Z_1^2(Z_2^2g_{xx}^2+\lambda^2B^2)}},\quad \Longrightarrow g_{tt}g_{rr}Z_2^2-\lambda^2A_t'^2=\frac{g_{tt}g_{rr}(Z_2^2g_{xx}^2+\lambda^2B^2)Z_1^2Z_2^2}{\rho^2+Z_1^2(Z_2^2g_{xx}^2+\lambda^2B^2)}. \tag{59}$$

With a non-trivial magnetic and electric field the equation of motion of the metric components gets modified and are given as

$$R_{tt}+\frac{V+2\Lambda}{2}g_{tt}+\frac{T_b}{2}\frac{Z_1^2Z_2^2g_{tt}g_{xx}}{\sqrt{\rho^2+Z_1^2(Z_2^2g_{xx}^2+\lambda^2B^2)}}=0, \tag{60}$$

$$R_{ij}-\frac{V+2\Lambda}{2}g_{xx}\delta_{ij}-T_b\frac{\delta_{ij}}{2}Z_2\sqrt{\rho^2+Z_1^2(Z_2^2g_{xx}^2+\lambda^2B^2)}=0, \tag{61}$$

$$R_{rr}-\frac{V+2\Lambda}{2}g_{rr}-\frac{1}{2}\phi'^2-\frac{T_b}{2}Z_1^2Z_2^3\frac{g_{rr}g_{xx}}{\sqrt{\rho^2+Z_1^2(Z_2^2g_{xx}^2+\lambda^2B^2)}}=0. \tag{62}$$

Now, we shall consider a specific configuration for which the potential energy is taken as trivial, $V=0$ and $Z_1=1=Z_2$. In this case, again, the trivial dilaton profile is a solution, $\phi=0$, to the equation of motion. In order to find the black hole solution, let us demand that the solution asymptotically looks as an AdS spacetime. In which case, the following ansatz to the metric solves the equations of motion

$$ds_{3+1}^2=\frac{r^2}{R^2}\Big[-f(r)dt^2+dx^2+dy^2\Big]+\frac{R^2}{r^2}\frac{dr^2}{f(r)}, \tag{63}$$

with some form for the function $f(r)$. From the g_{tt} and the g_{rr} part of the metric components we find the following second order differential equation for the function $f(r)$

$$r^2f''(r)+6rf(r)+6f(r)+2R^2\Lambda+\frac{T_br^2R^2}{\sqrt{r^4+R^4(\rho^2+\lambda^2B^2)}}=0, \tag{64}$$

whereas from the g_{xx} component of the metric, we find the following first order differential equation for $f(r)$

$$rf'(r)+3f(r)+\Lambda R^2+\frac{T_bR^2}{2r^2}\sqrt{r^4+R^4(\rho^2+\lambda^2B^2)}=0. \tag{65}$$

One can easily check that these two differential equations are not independent of each other. On solving the first order differential equation, we find the function, $f(r)$, has the following form

$$f(r)=\frac{c_1}{r^3}-\frac{\Lambda R^2}{3}-T_bR^2\frac{\sqrt{r^4+R^4(\rho^2+\lambda^2B^2)}}{6r^2}-\frac{T_bR^4\sqrt{\rho^2+\lambda^2B^2}}{3r^2}{}_2F_1\Big[\frac{1}{4},\frac{1}{2},\frac{5}{4},-\frac{r^4}{R^4(\rho^2+\lambda^2B^2)}\Big]. \tag{66}$$

It is easy to see that this solution in the zero magnetic field limit reproduces eq(45). As was done for the solution eq(45), we can identify the constant $c_1 \propto -M$, as the mass of the black hole. The horizon is determined from the zero's of the function, $f(r_h) = 0$ and the entropy density is given as $s = (2\pi/\kappa^2)(r_h/R)^2$.

The temperature of such a dyonic solution is

$$T_H = -\frac{r_h}{8\pi R^2}\left[2\Lambda + \frac{T_b}{r_h^2}\sqrt{(\rho^2+\lambda^2 B^2)R^4 + r_h^4}\right], \tag{67}$$

and the entropy in the extremal limit is

$$s_{ext} = \frac{2\pi}{\kappa^2}\frac{T_b\sqrt{\rho^2+\lambda^2 B^2}}{\sqrt{4\Lambda^2 - T_b^2}} \quad \neq 0. \tag{68}$$

There arises an interesting question: Is it possible that $4\Lambda^2 = T_b^2$? In which case the entropy in the extremal limit diverges. In order to answer such a question, let us look at the relation between the cosmological constant and the tension of the brane. From the condition of the vanishing temperature, there follows

$$\frac{4\Lambda^2}{T_b^2} = 1 + \frac{R^4(\rho^2+\lambda^2 B^2)}{r_h^4} \quad \Longrightarrow \quad \frac{4\Lambda^2}{T_b^2} \quad > \quad 1. \tag{69}$$

So, in the vanishing temperature limit the magnitude of the cosmological constant should be bigger than half the tension of the brane.

The chemical potential for such a dyonic solution is determined as

$$\begin{aligned}\mu &= \int_{r_h}^{\infty} dr\ A_t' = \frac{1}{\lambda}\int_{r_h}^{\infty} dr \frac{\rho R^2}{\sqrt{r^4 + (\rho^2+\lambda^2 B^2)R^4}} \\ &= \frac{\rho R^2}{\lambda}\left(\frac{4}{R(\rho^2+\lambda^2 B^2)^{1/4}\sqrt{\pi}}\Gamma^2\left(\frac{5}{4}\right) - r_h\ {}_2F_1\left[\frac{1}{2},\frac{1}{4},\frac{5}{4},-\frac{r_h^4}{R^4(\rho^2+\lambda^2 B^2)}\right]\right),\end{aligned} \tag{70}$$

4.2 Connection with the BI black hole

Recently, an electrically charged black hole solution is found in arbitrary spacetime dimension with the Born-Infeld (BI) matter [46], see also [47]- [53]. It is certainly interesting to find the connection between the Einstein-DBI black hole solutions for trivial dilaton with that of the Einstein-BI black holes in the presence of a cosmological constant. The BI matter and the DBI matter is described

$$S_{BI} = \int \sqrt{-g}\sqrt{1+\alpha F^{MN}F_{MN}}, \quad S_{DBI} = \int \sqrt{-det(g+\lambda F)_{MN}} \tag{71}$$

where α is a parameter. For small α we see the Maxwellian matter.

Generically, the DBI action is completely different from the BI action[2], but in a specific situation they can coincide. This happen only when the U(1) field strength has got one non-vanishing component. Let us illustrate this point by considering two non-vanishing component of the field strength, $F = A'_t(r)dr\wedge dt + Bdx\wedge dy$, in $d+1$ dimensional spacetime, $ds^2_{d+1} = -g_{tt}(r)dt^2 + g_{xx}(r)(dx^2+dy^2) + g_{xx}(r)dz_i^2 + g_{rr}(r)dr^2$.

On computing the BI and the DBI matter

$$\begin{aligned} S_{BI} &= \int \sqrt{g_{tt}g_{rr}}\, g_{xx}^{\frac{d-1}{2}} \sqrt{1 - 2\alpha\frac{A'^2_t}{g_{tt}g_{rr}} + 2\alpha\frac{B^2}{g^2_{xx}}}, \\ S_{DBI} &= \int \sqrt{g_{tt}g_{rr}}\, g_{xx}^{\frac{d-1}{2}} \sqrt{1 - \frac{\lambda^2 A'^2_t}{g_{tt}g_{rr}} + \frac{\lambda^2 B^2}{g^2_{xx}} - \frac{\lambda^4 B^2 A'^2_t}{g_{tt}g_{rr}g^2_{xx}}}. \end{aligned} \tag{72}$$

So, there follows that either for zero magnetic field or for zero charge density, $A'_t = 0$, both BI and DBI gives the same action for $2\alpha = \lambda^2$. Hence, its only the electrically charged black hole solution is same as found in [46], but not the dyonic solution.

5 Application: DC conductivity

As an application of the solutions found in the previous sections, we shall study various properties of the action as written in eq(7). To begin with, we shall calculate the dc conductivity. To do the computations, we shall use the flow equation technique of [45], which is done in [54] and [55] for the DBI system. In order to do the computations, let us first fluctuate both the metric and the gauge field components, i.e., g_{tx} and A_x and are assumed to be functions of time, t and r. The time dependence comes e.g. for the metric fluctuation as $g_{tx}(r)e^{-i\omega t}$. In what follows, we shall not be computing the conductivity for those solutions which has a non-zero magnetic field. Doing the above mentioned fluctuations, we find that the $x-r$ component of the equation of motion as written in eq(8) gives the following relation between the metric fluctuation and the gauge field fluctuation

$$\sqrt{g_{tt}g_{rr}Z_2^2 - \lambda^2 A'^2_t}\,(g'_{xx}g_{tx} - g_{xx}g'_{tx}) + \lambda^2 T_b Z_1 Z_2^{\frac{d-1}{2}} g_{xx}A_x A'_t\sqrt{g_{tt}g_{rr}} = 0, \tag{73}$$

where prime denotes derivative with respect to r and we have done the Fourier transformation with respect to $e^{-i\omega t}$, means we have set the momentum to zero. Also, in doing the computation, we have used the following result to the Ricci tensor $R_{xr} = \frac{i\omega}{2g_{tt}g_{xx}}(g'_{xx}g_{tx} - g_{xx}g'_{tx})$.

[2]Upon expanding the determinant in the DBI matter for arbitrary spacetime dimension $det(g+\lambda F)_{MN} = det(g)_{MN} + det(\lambda F)_{MN} + \cdots$, where the ellipses stands for various even powers of F. Upon restricting to $3+1$ dimensional spacetime, the $det(F)_{MN} \propto F\wedge F$ and this term is absent in the action of the BI matter.

Let us expand the gauge field part of the action as written in eq(7) to quadratic order in the gauge field, A_x, using eq(73) results in

$$S_A^{(2)} = -\frac{\lambda^2 T_b}{4\kappa^2}\int \frac{\sqrt{\rho^2 + Z_1^2 Z_2^{d-1} g_{xx}^{d-1}}}{Z_2 g_{xx}\sqrt{g_{tt}g_{rr}}}\left[g_{tt}A_x'^2 - A_x^2\left(\omega^2 g_{rr} + \frac{2\lambda^2 T_b A_t'^2\sqrt{\rho^2 + Z_1^2 Z_2^{d-1} g_{xx}^{d-1}}}{Z_2 g_{xx}^{\frac{d-1}{2}}}\right)\right]. \tag{74}$$

The equation of motion that follows from it takes the following form

$$\partial_r\left[\frac{\sqrt{\rho^2 + Z_1^2 Z_2^{d-1} g_{xx}^{d-1}}}{Z_2 g_{xx}\sqrt{g_{tt}g_{rr}}} g_{tt}A_x'\right] + \frac{\sqrt{\rho^2 + Z_1^2 Z_2^{d-1} g_{xx}^{d-1}}}{Z_2 g_{xx}\sqrt{g_{tt}g_{rr}}}\left(\omega^2 g_{rr} + \frac{2\lambda^2 T_b A_t'^2\sqrt{\rho^2 + Z_1^2 Z_2^{d-1} g_{xx}^{d-1}}}{Z_2 g_{xx}^{\frac{d-1}{2}}}\right) A_x = 0. \tag{75}$$

Let us compute the current at some choice of the radial coordinate, $r = r_c$, from eq(74)

$$J^x(r_c) = -\frac{\lambda^2 T_b}{2\kappa^2}\left[\frac{\sqrt{\rho^2 + Z_1^2 Z_2^{d-1} g_{xx}^{d-1}}}{Z_2 g_{xx}\sqrt{g_{tt}g_{rr}}} g_{tt}A_x'\right]_{r=r_c}. \tag{76}$$

Now assuming that the Ohm's law holds at any choice of the the radial slice, r_c, gives

$$J^x(r_c) = \sigma^{xx}(r_c,\omega)E_x(r_c) = \sigma^{xx}(r_c,\omega) i\omega A_x(r_c), \tag{77}$$

where in the second equality we have expressed all the quantities in the Fourier space. In order to see such a form of the Ohm's law, we assume that the retarded correlator at any radial slice is given, by generalizing eq(29) of [45]

$$G_R(r_c, k_\mu) = -\frac{\Pi(r_c, k_\mu)}{A_x(r_c, k_\mu)}, \tag{78}$$

where $\Pi(r_c, k_\mu)$ is the momentum associated to the field A_x, evaluated at r_c. In this case, the transport quantity, which is the conductivity, at r_c is related to the retarded correlator as $\sigma(r_c, k_\mu) = iG_R(r_c, k_\mu)/\omega = -i\Pi(r_c, k_\mu)/A_x(r_c, k_\mu)$. Note that the current in eq(76) is nothing but the momentum associated to A_x, hence there follows the Ohm's law at slice r_c, eq(77).

Let us evaluate the flow equation of the conductivity as we change the slice from r_c to $r_c + \delta r_c$ in the limit $\delta r_c \to 0$. In which case, the resulting flow equation becomes

$$\partial_{r_c}\sigma^{xx}(r_c,\omega) = -i\omega\sqrt{\frac{g_{rr}(r_c)}{g_{tt}(r_c)}}\left[\Sigma_A(r_c) - \frac{(\sigma^{xx}(r_c,\omega))^2}{\Sigma_A(r_c)} + \frac{4\lambda^2 T_b^2\rho^2}{4\kappa^2}\frac{Z_2(r_c)g_{tt}(r_c)}{g_{xx}^{\frac{d+1}{2}}(r_c)}\right], \tag{79}$$

where $\Sigma_A = 2\frac{\lambda^2 T_b}{4\kappa^2}\frac{\sqrt{\rho^2 + Z_1^2 z_2^{d-1} g_{xx}^{d-1}}}{Z_2 g_{xx}}$. At the horizon, the time component of the metric vanishes, $g_{tt}(r_h) = 0$, which means as we take the limit $r_c \to r_h$, we need to impose a regularity

condition on the conductivity at the horizon and the condition reads

$$\sigma^{xx}(r_h) = \Sigma_A(r_h) = 2\frac{\lambda^2 T_b}{4\kappa^2}\left[\frac{\sqrt{\rho^2 + Z_1^2 Z_2^{d-1} g_{xx}^{d-1}}}{Z_2 g_{xx}}\right]_{r_h}. \tag{80}$$

It is interesting to note that at the horizon the in-falling boundary condition for the gauge field, A_x, follows naturally combining the form of the conductivity at the horizon eq(80) and the Ohm's law at the horizon, i.e., $J^x(r_h) = \sigma^{xx}(r_h) i\omega A_x(r_h)$. In order to see it, let us use eq(76) and eq(80) in the Ohm's law. Then it follows that the derivative of the gauge field is related to the gauge field at the horizon as

$$A'_x(r_h) = -i\omega\left[\sqrt{\frac{g_{rr}}{g_{tt}}} A_x\right]_{r_h}. \tag{81}$$

Integrating this gives us the desired in-falling form of the gauge field at the horizon, namely

$$A_x(r_h) = e^{-i\omega \int^{r_h} dr \sqrt{\frac{g_{rr}}{g_{tt}}}}. \tag{82}$$

It is very easy to convince that in the zero frequency limit, i.e., the DC conductivity remains same over the entire range of the radial coordinate, which suggests that it does not run. Now given the form of the DC conductivity as in eq(80), there follows the following temperature dependence for different solutions

$$\sigma^{xx} = \frac{\lambda^2 T_b}{2\kappa^2}\sqrt{1+\rho^2} \times \begin{cases} (\frac{z+2}{4\pi})^{(2/z)}\ T_H^{-\frac{2}{z}} & \text{for eq(28) with } \phi_0 = 0 \\ (\frac{z+2}{4\pi})^{4/z}\ T_H^{-4/z} & \text{for eq(36).} \end{cases} \tag{83}$$

Let us recall that the longitudinal conductivity for the non-Fermi liquid (NFL) state of the matter has the inverse temperature dependence. Now at IR, we do generate such a behavior of the conductivity, if we choose the dynamical exponent $z = 2$ for the solution eq(28) whereas $z = 4$ for the solution written in eq(36). Such a behavior of the longitudinal conductivity has been found in [63] for $z = 2$, in the probe brane approximation. However, for $z = 2$, we find that the solution as written in eq(36) has the longitudinal conductivity, $\sigma^{xx} \sim T_H^{-2}$ and that of eq(28) has the longitudinal conductivity, $\sigma^{xx} \sim T_H^{-1}$. This implies that as far as the conductivity is concerned, by tuning the parameters like α and β, we can describe either the FL or the NFL state in a $2+1$ dimensional field theory. Such a crossover was found in a $3+1$ dimensional field theory using a magnetic field in [55].

6 Entanglement entropy

The entanglement entropy of a d dimensional field theoretic system or a $d+1$ dimensional gravitational system is determined by finding the $d-1$ dimensional minimal spacelike hypersurface, γ_A, that extremizes the area of the hypersurface [56]. The explicit formula for

the entanglement entropy is $S_A = \frac{\text{Area of } \gamma_a}{4G_N}$, where G_N is the Newton's constant in $d+1$ dimensional gravitational system. Various aspects of the entanglement entropy is further studied in [57]-[62].

For the geometry of the form as written in eq(11), the geometry of the $d-1$ dimensional hypersurface takes the following form

$$ds^2_{d-1} = \left[g_{rr}(r)\left(\frac{dr}{dx_1}\right)^2 + g_{xx}(r)\right]dx_1^2 + g_{xx}(r)(dx_2^2 + \cdots + dx_{d-1}^2), \tag{84}$$

where the precise nature of the hypersurface is determined by the function $r(x_1)$. On computing the area of this hypersurface, γ_A, we find

$$\mathcal{A}(\gamma_A) = \int dx_2 \cdots \int dx_{d-1} \int dr\ g_{xx}^{\frac{d-2}{2}}\ \sqrt{g_{rr} + g_{xx}(dx_1/dr)^2}. \tag{85}$$

In order to carry out the integral of x_2 to x_{d-1}, let us assume for simplicity the hypersurface has the shape of a strip. In which case, we assume that $-\ell \le x_1 \le \ell$ and $0 \le (x_2, \cdots, x_{d-1}) \le L$. Finally doing the integrals results in

$$\mathcal{A}(\gamma_A) = L^{d-2}\int_{-\ell}^{\ell} dx_1\ g_{xx}^{\frac{d-2}{2}}\ \sqrt{g_{xx} + g_{rr}(dr/dx_1)^2}. \tag{86}$$

Extremizing the surface area gives the following solution to the function $r(x_1)$

$$\frac{dr}{dx_1} = \frac{\sqrt{g_{xx}^d(r) - g_{xx}(r)g_{xx}^{d-1}(r_\star)}}{g_{xx}^{\frac{d-1}{2}}(r_\star)\sqrt{g_{rr}(r)}}, \tag{87}$$

where the turning point, $r_\star$, is determined from

$$\ell = \int_{r_\star}^{\infty} dr \frac{g_{xx}^{\frac{d-1}{2}}(r_\star)\sqrt{g_{rr}(r)}}{\sqrt{g_{xx}^d(r) - g_{xx}(r)g_{xx}^{d-1}(r_\star)}} = g_{xx}^{\frac{d-1}{2}}(r_\star)\int_{r_\star}^{\infty} dr\sqrt{\frac{g_{rr}(r)}{g_{xx}^d(r)}}\frac{1}{\sqrt{1-\frac{g_{xx}^{d-1}(r_\star)}{g_{xx}^{d-1}(r)}}} \tag{88}$$

Finally, substituting this form of the function, $r(x_1)$, from eq(87) into the area gives

$$\mathcal{A}(\gamma_A) = L^{d-2}\int_{r_\star}^{\Lambda} dr\ \frac{g_{xx}^{d-1}(r)\sqrt{g_{rr}(r)}}{\sqrt{g_{xx}^d(r) - g_{xx}(r)g_{xx}^{d-1}(r_\star)}} = L^{d-2}\int_{r_\star}^{\Lambda} dr\ \frac{\sqrt{g_{xx}^{d-2}(r)g_{rr}(r)}}{\sqrt{1-\frac{g_{xx}^{d-1}(r_\star)}{g_{xx}^{d-1}(r)}}}, \tag{89}$$

where Λ is the UV-cutoff which will regulate the presence of the divergence while taking the $r \to \infty$ limit. For trivial hypersurfaces, which are defined as $dr/dx_1 = 0$, the area can't be computed using eq(89). Rather, we should use eq(86) to do the calculations. Recall that

for finite metric components, $dr/dx_1 = 0$ means the spatial part of the metric components are constants, i.e., $g_{xx}(r) = d_0$=constant. In which case the entanglement entropy precisely matches with the Bekenstein-Hawking entropy and this happens for spacetimes which are of the $AdS_2 \times R^{d-1}$ type.

It is argued in [22] that the presence of the logarithmic structure in the entanglement entropy suggests the presence of Fermi surfaces. There are two ways to generate such a logarithmic structure to the area (a) from eq(89), whenever the metric components satisfy, $g_{rr} g_{xx}^{d-2} = 1/r^2$, with a power law type of relation between ℓ and the turning point $r_\star$ (b) from eq(88), with ℓ is related to the turning point $r_\star$ logarithmically, i.e., $r_\star \sim Log\ \ell$, and from eq(89), the area should come as a power law in $r_\star$.

We can easily see the entanglement entropy that goes logarithmically for the type (a) are $d = 2$, with $g_{rr} = 1/r^2$ (constant/non-constant metric components of, $g_{xx}(r)$), [56]. However, for $d > 2$ with $g_{xx} = r^2$ means g_{rr} should be as $g_{rr} = r^{-2(d-1)}$. One such example is studied in [22] for $d = 3$.

For type (b) the simplest choice is to take the spatial component of the metric to go exponentially. In which case, it could be possible that the curvature is diverging.

The examples that are studied in the present paper does not show the presence of the logarithmic structure in the expression of the entanglement entropy when the entangling region is of the strip type with the dimensionality of the bulk spacetime four, i.e., $d = 3$.

7 Conclusion

In this paper, we have found the geometry created by considering the back reaction of a space filling D-brane in the presence of a scalar field, i.e., new solutions to Einstein-DBI-dilaton system. The most notable solution shows Fermi-like liquid behavior in $3 + 1$ dimensional gravitational description. This we demonstrate by finding a black hole solution which asymptotes to Lifshitz spacetime. For a specific choice of the dynamical exponent, $z = 2$, the longitudinal conductivity and the specific heat goes as inverse quadratic and linear in temperature, respectively. Even though the geometry and the U(1) gauge field is scaling invariant but the presence of a non-trivial dilaton profile breaks it. At IR, the field strength vanishes whereas the dilaton diverges. We leave the study of the dispersion relation obeyed by these solutions, along the lines of [44] and [67], for future research.

In the absence of the dilaton, we found an electrically charged black hole solution in any arbitrary spacetime dimensions as well as a dyonic black hole solution in $3 + 1$ dimensional spacetime. Its only the electrically charged black hole solution are same as that obtained for the BI black holes in [46]. Interestingly, these solutions asymptotes to AdS spacetime. These black holes, without dilaton, have some non-zero entropy even in the zero temperature limit. Whereas the entropy for the black holes with dilaton vanishes as the temperature vanishes. We leave the detailed study of the thermodynamics for future investigations. Instead of

considering the space filling Dd-brane action, it would be interesting to study the lower dimensional brane action along the lines of [65] with massive embeddings [66], whose solution as well as the rich thermodynamics, we leave for future research.

Acknowledgment
It is a pleasure to thank Bum-Hoon Lee, Shibaji Roy and Tadashi Takayanagi for their useful comments, especially to Elias Kiritsis for several useful comments and suggestions. Thanks are to Saha institute of Nuclear Physics, Kolkata and CQUeST, Sogang university, Seoul for their generous help and support at different stages of this work.

References

[1] C. Charmousis, B. Gouteraux, B. S. Kim, E. Kiritsis and R. Meyer, JHEP **1011**, 151 (2010), [arXiv:1005.4690 [hep-th]].

[2] B. Gouteraux and E. Kiritsis, JHEP **1112**, 036 (2011), [arXiv:1107.2116 [hep-th]].

[3] L. Huijse, S. Sachdev and B. Swingle, Phys. Rev. B **85**, 035121 (2012), [arXiv:1112.0573 [cond-mat.str-el]].

[4] J. M. Maldacena, Adv. Theor. Math. Phys. **2**, 231 (1998) [hep-th/9711200]; S. S. Gubser, I. R. Klebanov and A. M. Polyakov, Phys. Lett. **B 428** (1998) 105, [arXiv:hep-th/9802109]; E. Wittwn, Adv. Theor. Math. Phys. **2**, 253 (1998), [arXiv:hep-th/9802150]; O. Aharony, S. S. Gubser, J. Maldacena, H. Ooguri and Y. Oz, Phys. Rept, **323**, 183-386 (2000), [arXiv:hep-th/9905111].

[5] X. Dong, S. Harrison, S. Kachru, G. Torroba and H. Wang, JHEP **1206**, 041 (2012), [arXiv:1201.1905 [hep-th]].

[6] K. Narayan, Phys. Rev. D **85**, 106006 (2012), [arXiv:1202.5935 [hep-th]].

[7] B. S. Kim, JHEP **1206**, 116 (2012), [arXiv:1202.6062 [hep-th]].

[8] H. Singh, JHEP **1207**, 082 (2012), [arXiv:1202.6533 [hep-th]].

[9] P. Dey and S. Roy, [arXiv:1203.5381 [hep-th]]; P. Dey and S. Roy, [arXiv:1204.4858 [hep-th]].

[10] E. Perlmutter, JHEP **1206**, 165 (2012), [arXiv:1205.0242 [hep-th]].

[11] M. Ammon, M. Kaminski and A. Karch, [arXiv:1207.1726 [hep-th]].

[12] M. Alishahiha and H. Yavartanoo, [arXiv:1208.6197 [hep-th]].

[13] M. Alishahiha, E. O Colgain and H. Yavartanoo, [arXiv:1209.3946 [hep-th]].

[14] S. Kachru, X. Liu and M. Mulligan, Phys. Rev. D **78**, 106005 (2008) [arXiv:0808.1725 [hep-th]].

[15] M. Taylor, [arXiv:0812.0530 [hep-th]].

[16] S. S. Pal, [arXiv:0901.0599 [hep-th]].

[17] S. Kachru, R. Kallosh and M. Shmakova, Phys. Rev. D **84**, 046003 (2011), [arXiv:1104.2884 [hep-th]].

[18] K. Inbasekar and P. K. Tripathy, JHEP **1209**, 003 (2012), [arXiv:1206.3887 [hep-th]].

[19] S. S. Gubser and F. D. Rocha, Phys. Rev. D **81**, 046001 (2010), [arXiv:0911.2898 [hep-th]].

[20] K. Goldstein, S. Kachru, S. Prakash and S. P. Trivedi, JHEP **1008**, 078 (2010), [arXiv:0911.3586 [hep-th]].

[21] C. -M. Chen and D. -W. Pang, JHEP **1006**, 093 (2010), [arXiv:1003.5064 [hep-th]].

[22] N. Ogawa, T. Takayanagi and T. Ugajin, JHEP **1201**, 125 (2012), [arXiv:1111.1023 [hep-th]].

[23] S. Sachdev, Phys. Rev. Lett. **105**, 151602 (2010), [arXiv:1006.3794 [hep-th]].

[24] S. Sachdev, Phys. Rev. D **84**, 066009 (2011), [arXiv:1107.5321 [hep-th]].

[25] K. Jensen, S. Kachru, A. Karch, J. Polchinski and E. Silverstein, Phys. Rev. D **84**, 126002 (2011), [arXiv:1105.1772 [hep-th]].

[26] S. A. Hartnoll and A. Tavanfar, Phys. Rev. D **83**, 046003 (2011), [arXiv:1008.2828 [hep-th]]; S. A. Hartnoll and P. Petrov, Phys. Rev. Lett. 106, 121601 (2011), [arXiv:1011.6469[hep-th]].

[27] S. -S. Lee, Phys. Rev. D **79**, 086006 (2009), [arXiv:0809.3402 [hep-th]].

[28] H. Liu, J. McGreevy and D. Vegh, Phys. Rev. D **83**, 065029 (2011), [arXiv:0903.2477 [hep-th]].

[29] T. Faulkner, H. Liu, J. McGreevy and D. Vegh, Phys. Rev. D **83**, 125002 (2011), [arXiv:0907.2694 [hep-th]].

[30] M. Cubrovic, J. Zaanen and K. Schalm, Science **325**, 439 (2009), [arXiv:0904.1993 [hep-th]].

[31] N. Iizuka, N. Kundu, P. Narayan and S. P. Trivedi, JHEP **1201**, 094 (2012), [arXiv:1105.1162 [hep-th]].

[32] R. Belliard, S. S. Gubser and A. Yarom, JHEP **1110**, 055 (2011), [arXiv:1106.6030 [hep-th]].

[33] J. P. Gauntlett, J. Sonner and D. Waldram, Phys. Rev. Lett. **107**, 241601 (2011), [arXiv:1106.4694 [hep-th]].

[34] O. DeWolfe, S. S. Gubser and C. Rosen, Phys. Rev. Lett. **108**, 251601 (2012), [arXiv:1112.3036 [hep-th]]; [arXiv:1207.3352[hep-th]].

[35] D. Guarrera and J. McGreevy, [arXiv:1102.3908 [hep-th]].

[36] J. N. Laia and D. Tong, JHEP **1111**, 125 (2011), [arXiv:1108.1381 [hep-th]].

[37] W. -J. Li, R. Meyer and H. -b. Zhang, JHEP **1201**, 153 (2012), [arXiv:1111.3783 [hep-th]].

[38] N. Iqbal, H. Liu and M. Mezei, [arXiv:1110.3814 [hep-th]].

[39] R. Gopakumar, A. Hashimoto, I. R. Klebanov, S. Sachdev and K. Schoutens, [arXiv:1206.4719 [hep-th]].

[40] U. Gursoy, V. Jacobs, E. Plauschinn, H. Stoof and S. Vandoren, [arXiv:1209.2593 [hep-th]].

[41] A. Salvio, [arXiv:1207.3800 [hep-th]].

[42] O. Bergman, N. Jokela, G. Lifschytz and M. Lippert, JHEP **1110**, 034 (2011), [arXiv:1106.3883 [hep-th]].

[43] M. Kulaxizi and A. Parnachev, Phys. Rev. D **78**, 086004 (2008), [arXiv:0808.3953 [hep-th]].

[44] A. Karch, D. T. Son and A. O. Starinets, [arXiv:0806.3796 [hep-th]].

[45] N. Iqbal and H. Liu, Phys. Rev. D **79**, 025023 (2009), [arXiv:0809.3808 [hep-th]].

[46] T. K. Dey, Phys. Lett. B **595**, 484 (2004), [hep-th/0406169]; M. Cataldo and A. Garcia, Phys. Lett. B **456**, 28 (1999), [hep-th/9903257].

[47] S. Fernando and D. Krug, Gen. Rel. Grav. **35**, 129 (2003), [hep-th/0306120];

[48] R. -G. Cai, D. -W. Pang and A. Wang, Phys. Rev. D **70**, 124034 (2004), [hep-th/0410158].

[49] A. Sheykhi and N. Riazi, Phys. Rev. D **75**, 024021 (2007), [hep-th/0610085].

[50] S. Fernando, Phys. Rev. D **74**, 104032 (2006), [hep-th/0608040].

[51] Y. S. Myung, Y. -W. Kim and Y. -J. Park, Phys. Rev. D **78**, 044020 (2008), [arXiv:0804.0301 [gr-qc]]; Phys. Rev. D **78**, 084002 (2008), [arXiv:0805.0187 [gr-qc]].

[52] S. Banerjee, Phys. Rev. D **82**, 106008 (2010), [arXiv:1009.1780 [hep-th]].

[53] Y. S. Myung, and T. Moon, [arXiv:1201.1173 [gr-qc]].

[54] B. -H. Lee, S. S. Pal and S. -J. Sin, Int. J. Mod. Phys. A **27**, 1250071 (2012), [arXiv:1108.5577 [hep-th]].

[55] S. S. Pal, [arXiv:1202.3555 [hep-th]].

[56] S. Ryu and T. Takayanagi, Phys. Rev. Lett. **96**, 181602 (2006), [arXiv:hep-th/0603001]; S. Ryu and T. Takayanagi, JHEP 0608, 045 (2006), [arXiv:hep-th/0605073].

[57] W. Li, T. Nishioka, T. Takayanagi, JHEP, **0910** (2009) 015, [arXiv:0908.0363[hep-th]].

[58] M. Headrick, Phys. Rev. D **82**, 126010 (2010), [arXiv:1006.0047 [hep-th]].

[59] J. de Boer, M. Kulaxizi and A. Parnachev, JHEP **1107**, 109 (2011), [arXiv:1101.5781 [hep-th]].

[60] H. Casini, M. Huerta and R. C. Myers, JHEP **1105**, 036 (2011), [arXiv:1102.0440 [hep-th]].

[61] R. C. Myers and A. Singh, [arXiv:1206.5225[hep-th]].

[62] M. Kulaxizi, A. Parnachev and K. Schalm, [arXiv:1208.2937 [hep-th]].

[63] S. A. Hartnoll, J. Polchinski, E. Silverstein and D. Tong, JHEP **1004**, 120 (2010), [arXiv:0912.1061 [hep-th]].

[64] M. Jarvinen and E. Kiritsis, JHEP **1203**, 002 (2012), [arXiv:1112.1261 [hep-ph]].

[65] S. B. Giddings, S. Kachru and J. Polchinski, Phys. Rev. D **66**, 106006 (2002), [hep-th/0105097].

[66] D. Mateos, R. C. Myers and R. M. Thomson, Phys. Rev. Lett. **98**, 101601 (2007), [hep-th/0610184].

[67] D. K. Brattan, R. A. Davison, S. A. Gentle and A. O'Bannon, [arXiv:1209.0009 [hep-th]].